

Logarithm and Its Applications

DPP 1.1

Exponential and Logarithmic Functions

Single Correct Answer Type

- If $x \in N$, then the value of x satisfying the equation $5^x \cdot \sqrt[3]{8^{x-1}} = 500$ is divisible by
(a) 2 (b) 4 (c) 3 (d) 5
- If $\log_{175} 5x = \log_{343} 7x$, then the value of $\log_{42}(x^4 - 2x^2 + 7)$ is
(a) 1 (b) 2 (c) 3 (d) 4
- The value of $\log_{10}(\sqrt{3-\sqrt{5}} + \sqrt{3+\sqrt{5}})$ is
(a) $1/2$ (b) $1/4$
(c) $3/2$ (d) $3/4$
- Which of the following is not the solution of $\frac{16^{1/x}}{2^{x+3}} > 1$?
(a) $(-\infty, -4)$ (b) $(0, 1)$
(c) $(0, \infty)$ (d) none of these
- If $\log_4 A = \log_6 B = \log_9(A+B)$ then the value of $\frac{B}{A}$ is
(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{4}$
(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{5}+1}{2}$
- The value of $\log_{\frac{9}{4}} \left(\frac{1}{2\sqrt{3}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \dots \infty \right)$ is
(a) -2 (b) -1
(c) -1/2 (d) none of these
- Number of real values of x satisfying the equation $\log_{x^2+6x+8}(\log_{2x^2+2x+3}(x^2-2x)) = 0$ is equal to
(a) 3 (b) 2 (c) 1 (d) 0
- $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q(\log_r(\log_p x)) = 0$, then 'p' equals
(a) $r^{q/r}$ (b) rq (c) 1 (d) $r^{r/q}$

Answers Key

Single Correct Answer Type

1. (c) 2. (a) 3. (a) 4. (c) 5. (d)

6. (c) 7. (c) 8. (a)

DPP 1.2

Properties of Logarithm

Single Correct Answer Type

- The greatest integer less than or equal to the number $\log_2 15 \times \log_{1/6} 2 \times \log_3 1/6$ is
(a) 4 (b) 3 (c) 2 (d) 1
- Given that $\log_2 3 = a$, $\log_3 5 = b$, $\log_7 2 = c$, then the value of $\log_{140} 63$ is equal to
(a) $\frac{2+ac}{2c+1+abc}$ (b) $\frac{1+2ac}{c+2+abc}$
(c) $\frac{1+2ac}{2c+1+abc}$ (d) $\frac{2+ac}{c+2+abc}$
- If $\frac{\log_2 x}{4} = \frac{\log_2 y}{6} = \frac{\log_2 z}{3k}$ and $x^3 y^2 z = 1$, then k is equal to
(a) -8 (b) -4 (c) 0 (d) 4
- A line $x = k$ intersects the graph of $y = \log_4 x$ and $y = \log_4(x+4)$. The distance between the points of intersection is 0.5, then the value of k is
(a) 1 (b) 2 (c) 3 (d) 4
- The value of $\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$ is
(a) 1 (b) 2 (c) 3 (d) 4
- The value of $\frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} + \frac{1}{\log_{ab} abc}$ is equal to $(a, b, c > 0)$
(a) $1/2$ (b) 1 (c) 2 (d) 4
- $\log_a N \cdot \log_b N + \log_b N \cdot \log_c N + \log_c N \cdot \log_a N$ is equal to
(a) $\frac{\log_a N \cdot \log_b N \cdot \log_c N}{\log_{abc} N}$
(b) $\frac{\log_{abc} N}{\log_a N \cdot \log_b N \cdot \log_c N}$
(c) $\frac{\log_{Nabc}}{\log_N a \cdot \log_N b \cdot \log_N c}$
(d) none of these
- There exist positive integers A, B and C with no common factors greater than 1, such that $A \log_{200} 5 + B \log_{200} 2 = C$. The sum $A + B + C$ equals
(a) 5 (b) 6 (c) 7 (d) 8
- The value of $(2^{\log_6 18}) \cdot (3^{\log_6 3})$ is
(a) 6 (b) 9 (c) 12 (d) 18
- If $\log_a b = 2$, $\log_b c = 2$ and $\log_3 c = 3 + \log_3 a$, then the value of $\frac{c}{ab}$ is
(a) 1 (b) 3 (c) 9 (d) 27
- If $\log_2 10 = p$; $\frac{\log_e 10}{\log_e 7} = q$ and $(11)^r = 10$, then which one of the following expressions is equivalent to $\log_{10} 154$?
(a) pqr (b) $\frac{1}{pqr}$
(c) $\frac{p+q+r}{pqr}$ (d) $\frac{pq+qr+rp}{pqr}$
- If $3^{(\log_3 7)^x} = 7^{(\log_7 3)^x}$, then the value of x will be
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) 1
- The value of $4^{5 \log_{4\sqrt{2}} (3-\sqrt{6}) - 6 \log_8 (\sqrt{3}-\sqrt{2})}$ is
(a) 3 (b) 6 (c) 9 (d) 27
- The value of $\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409} \left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6} \right)$ is
(a) 0 (b) 1 (c) 2 (d) 3
- The value of $6^{\log_{10} 40} \cdot 5^{\log_{10} 36}$ is
(a) 200 (b) 216
(c) 432 (d) none of these

Answers Key

Single Correct Answer Type

- | | | | | | | | | | |
|--------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (c) | 3. (a) | 4. (d) | 5. (c) | 6. (c) | 7. (a) | 8. (b) | 9. (a) | 10. (b) |
| | | | | | 11. (d) | 12. (a) | 13. (c) | 14. (b) | 15. (b) |

DPP 1.3

Logarithmic Equations

Subjective Type

- Find the value of x satisfying the equations $\log_3(\log_2 x) + \log_{1/3}(\log_{1/2} y) = 1$ and $xy^2 = 9$
- Let a and b be real numbers greater than 1 for which there exists a positive real number c , different from 1, such that $2(\log_a c + \log_b c) = 9\log_{ab} c$. Find the largest possible value of $\log_a b$.
- Solve: $\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x$.
- Solve: $\frac{3}{2} \log_4(x+2)^2 + 3 = \log_4(4-x)^3 + \log_4(6+x)^3$
- Solve: $\log_{3/4} \log_8(x^2+7) + \log_{1/2} \log_{1/4}(x^2+7)^{-1} = -2$
- If $5^{\log x} - 3^{\log x-1} = 3^{\log x+1} - 5^{\log x-1}$, where the base of logarithm is 10, then find the value of x .
- Solve the system of equations:
 $(\log_a x)(\log_a(xyz)) = 48$
 $(\log_a y)(\log_a(xyz)) = 12$
 $(\log_a z)(\log_a(xyz)) = 84$
- Solve: $\sqrt[4]{|x-3|^{x+1}} = \sqrt[3]{|x-3|^{x-2}}$
- Solve: $\log_{x^2} 16 + \log_{2x} 64 = 3$

Single Correct Answer Type

- The value of x for which equation $5 \cdot 3^{\log_3 x} - 2^{1-\log_2 x} - 3 = 0$ is
 (a) 1 (b) 2 (c) $\frac{2}{5}$ (d) 7

- The sum of all values of a satisfying the equation

$$\begin{vmatrix} \log_{10} a & -1 \\ \log_{10}(a-1) & 2 \end{vmatrix} = \log_{10} a + \log_{10} 2$$
 is

- 0
 - 1
 - 2
 - none of these
- The number of real solution(s) of the equation $9^{\log_3(\log_e x)} = \log_e x - (\log_e x)^2 + 1$ is equal to
 (a) 0 (b) 1 (c) 2 (d) 3
 - The solution set of the equation $x^{\log_x(1-x)^2} = 9$, is
 (a) $\{-2, 4\}$ (b) $\{4\}$
 (c) $\{0, -2, -4\}$ (d) none of these
 - The value of x satisfying the equation
 $(\sqrt{\pi})^{\log_{\pi} x} \cdot (\sqrt{\pi})^{\log_{\pi^2} x} \cdot (\sqrt{\pi})^{\log_{\pi^4} x} \cdot (\sqrt{\pi})^{\log_{\pi^8} x} \dots \infty = 3$ is equal to
 (a) $\sqrt{\pi}$ (b) π (c) 3 (d) $\frac{1}{3}$
 - If $a > 1, x > 0$ and $2^{\log_a(2x)} = 5^{\log_a(5x)}$, then x is equal to
 (a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) 1
 - If x_1 and x_2 are solutions of the equation
 $\log_5 \left(\log_{64}|x| + (25)^x - \frac{1}{2} \right) = 2x$, then
 (a) $x_1 = 2x_2$ (b) $x_1 + x_2 = 0$
 (c) $x_1 = 3x_2$ (d) $x_1 x_2 = 64$
 - If $\log_6 9 - \log_9 27 + \log_8 x = \log_{64} x - \log_4 4$, then the value of x is
 (a) $1/2$ (b) $1/4$ (c) $1/8$ (d) $1/16$

Answers Key

Subjective Type

- $x = 729$
- 2
- 60
- $x = 2, 1 - \sqrt{33}$
- ± 3
- 100
- $x = a^4, y = a, z = a^7$
- $x = 4, 2; x = 11$
- $x = 2^{-1/3}, 4$

Single Correct Answer Type

- (a)
- (c)
- (b)
- (b)
- (c)
- (b)
- (c)

DPP 1.4

Inequalities and Finding Logarithm

Single Correct Answer Type

- If $\log_2(\log_2(\log_2 x)) = 2$, then the number of digits in x , is ($\log_{10} 2 = 0.3010$)
(a) 7 (b) 6 (c) 5 (d) 4
- The number of integers satisfying the inequality $\log_{\sqrt{0.9}} \log_5(\sqrt{x^2 + 5 + x}) > 0$ is
(a) 6 (b) 7 (c) 8 (d) 9
- The smallest integral x satisfying the inequality $\frac{1 - \log_4 x}{1 + \log_2 x} \leq \frac{1}{2}$ is
(a) $\sqrt{2}$ (b) 2 (c) 3 (d) 4
- The number of integral solutions of $\log_9(x+1) \cdot \log_2(x+1) - \log_9(x+1) - \log_2(x+1) + 1 < 0$ is
(a) 4 (b) 5 (c) 6 (d) 7
- The number of integers satisfying $\log_{\frac{1}{x}} \left(\frac{2(x-2)}{(x+1)(x-5)} \right) \geq 1$ is
(a) 0 (b) 1 (c) 2 (d) 3

Multiple Correct Answers Type

- If $\log_{1/2}(4-x) \geq \log_{1/2} 2 - \log_{1/2}(x-1)$, then x can belong to
(a) (1, 2] (b) [1, 3] (c) [3, 4] (d) [2, 3]
- The values of x satisfying $2 \log_{\frac{1}{4}}(x+5) > \frac{9}{4} \log_{\frac{1}{3\sqrt{3}}}(9) + \log_{\sqrt{x+5}}(2)$ is/are
(a) (-5, -4) (b) (-3, -1)
(c) (-4, -1) (d) (-5, -2)
- If $\log_3 x - (\log_3 x)^2 \leq \frac{3}{2} \log_{(1/2\sqrt{2})} 4$, then x can belong to
(a) $(-\infty, 1/3)$ (b) $(9, \infty)$
(c) (1, 6) (d) $(-\infty, 0)$
- Which of the following is/are true?
(a) number of digits in $8^{12} 5^{35}$ is 35
(b) number of digits in $8^{12} 5^{35}$ is 36
(c) number of zeroes after decimal before a significant figure starts in $\left(\frac{8}{27}\right)^{20}$ is 10
(d) number of zeroes after decimal before a significant figure starts in $\left(\frac{8}{27}\right)^{20}$ is 11

Answers Key

Single Correct Answer Type

1. (c) 2. (c) 3. (b) 4. (c) 5. (a)

Multiple Correct Answers Type

6. (a, c) 7. (a, b) 8. (a, b) 9. (b, c)

Trigonometric Functions

DPP 2.1

Trigonometric Ratios of Acute Angles; Trigonometric Identities

Single Correct Answer Type

- The circular wire of diameter 10 cm is cut and placed along the circumference of a circle of diameter 1 meter. The angle subtended by the wire at the center of circle is equal to
 - $\frac{\pi}{4}$ radian
 - $\frac{\pi}{3}$ radian
 - $\frac{\pi}{5}$ radian
 - $\frac{\pi}{10}$ radian
- $\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} =$
 - 0
 - 1
 - $\cos \theta - \sin \theta$
 - $\cos \theta + \sin \theta$
- If $\theta \in (\pi/4, \pi/2)$ and $\sum_{n=1}^{\infty} \frac{1}{\tan^n \theta} = \sin \theta + \cos \theta$, then the value of $\tan \theta$ is
 - $\sqrt{3}$
 - $\sqrt{2} + 1$
 - $2 + \sqrt{3}$
 - $\sqrt{2}$
- The value of $\frac{\tan^2 20^\circ - \sin^2 20^\circ}{\tan^2 20^\circ \cdot \sin^2 20^\circ}$ is
 - 1/2
 - 1
 - 2
 - none of these
- If $15 \sin^4 \alpha + 10 \cos^4 \alpha = 6$, then the value of $8 \operatorname{cosec}^6 \alpha + 27 \sec^6 \alpha$ is
 - 150
 - 175
 - 225
 - 250
- In a $\triangle ABC$, $\sin A + \sin B + \sin C = 1 + \sqrt{2}$ and $\cos A + \cos B + \cos C = \sqrt{2}$ if the triangle is
 - equilateral
 - isosceles
 - right angled
 - right angle isosceles
- If $\frac{\sin^2 x - 2 \cos^2 x + 1}{\sin^2 x + 2 \cos^2 x - 1} = 4$, then the value of $2 \tan^2 x$ is
 - 3
 - 4
 - 5
 - 6
- If $\sin \theta, \tan \theta, \cos \theta$ are in G.P., then $4 \sin^2 \theta - 3 \sin^4 \theta + \sin^6 \theta =$
 - 1
 - 2
 - 1
 - None of these
- If $\tan \theta - \cot \theta = a$ and $\sin \theta + \cos \theta = b$, then $(b^2 - 1)^2 (a^2 + 4)$ is equal to
 - 2
 - 4
 - ± 4
 - 4
- The least value of $18 \sin^2 \theta + 2 \operatorname{cosec}^2 \theta - 3$ is
 - 15
 - 12
 - 0
 - 9
- If $\tan^2 \alpha \tan^2 \beta + \tan^2 \beta \tan^2 \gamma + \tan^2 \gamma \tan^2 \alpha + 2 \tan^2 \alpha \tan^2 \beta \tan^2 \gamma = 1$, then the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ is
 - 0
 - 1
 - 1
 - none of these
- If x, y, z are all positive acute angles, then the least value of $\tan x (\cot y + \cot z) + \tan y (\cot z + \cot x) + \tan z (\cot x + \cot y)$, is
 - 2
 - 4
 - 6
 - 8

2.2 Trigonometry

13. Three circles, each of radius 1, touch one another externally and they lie between two parallel lines. The minimum possible distance between the lines is

(a) $2 + \sqrt{3}$ (b) $3 + \sqrt{3}$ (c) 4 (d) $2 + \frac{1}{\sqrt{3}}$

14. If $\frac{\cos \alpha}{\cos A} + \frac{\sin \alpha}{\sin A} = \frac{\cos \beta}{\cos A} + \frac{\sin \beta}{\sin A} = 1$, where α and β do not differ by an even multiple of π , prove that $\frac{\cos \alpha \cos \beta}{\cos^2 A} + \frac{\sin \alpha \sin \beta}{\sin^2 A} =$
- (a) -2 (b) -1 (c) 1 (d) 2

Answers Key

Single Correct Answer Type

1. (c) 2. (d) 3. (a) 4. (b) 5. (d)

6. (d)

7. (c)

8. (c)

9. (d)

10. (d)

11. (c)

12. (c)

13. (a)

14. (b)

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DPP 2.2

Trigonometric Functions

Single Correct Answer Type

- Consider angles $\alpha = \left(2n + \frac{1}{2}\right)\pi \pm A$ and $\beta = m\pi + (-1)^m \left(\frac{\pi}{2} - A\right)$ where $n, m \in I$. Which of the following is not true?
 - α and β are always the same angles
 - α and β are co-terminal angles
 - $\sin \alpha = \sin \beta$ but $\cos \alpha \neq \cos \beta$
 - none of these
- Which of the following is true?
 - $\sin 765^\circ = -\frac{1}{\sqrt{2}}$
 - $\cot\left(-\frac{15\pi}{4}\right) = -1$
 - $\tan \frac{13\pi}{3} = \frac{1}{\sqrt{3}}$
 - $\operatorname{cosec}(-1410^\circ) = 2$
- If the angle A of a triangle ABC is given by the equation $5 \cos A + 3 = 0$, then $\sin A$ and $\tan A$ are the roots of the equation
 - $15x^2 - 8x - 16 = 0$
 - $15x^2 - 8\sqrt{2}x + 16 = 0$
 - $15x^2 - 8x + 16 = 0$
 - $15x^2 + 8x - 16 = 0$
- Which of the following is greatest?
 - $\tan 1$
 - $\tan^2 1$
 - $\cot 1$
 - $\cot^2 1$
- The number of value/values of x for which $\sin y = x^2 - 2x$ is possible is
 - 0
 - 1
 - infinite
 - none of these
- Which of the following is not correct?
 - $\cos(\cos 1) > \cos(\sin 1)$
 - $\sin(\cos 1) < \sin(\sin 1)$
 - $\cos(\cos 2) > \sin(\cos 2)$
 - none of these
- If $\sin^4 \alpha + \cos^4 \beta + 2 = 4 \sin \alpha \cos \beta$, $0 \leq \alpha, \beta \leq \frac{\pi}{2}$, then $(\sin \alpha + \cos \beta)$ is equal to
 - $\sqrt{2}$
 - $\frac{1}{2}$
 - 2
 - 1
- The number of ordered pairs (a, x) satisfying the equation $\sec^2(a+2)x + a^2 - 1 = 0$; $-\pi < x < \pi$ is
 - 2
 - 1
 - 3
 - infinite
- The range of $y = \sin^3 x - 6 \sin^2 x + 11 \sin x - 6$ is
 - $[-24, 2]$
 - $[-24, 0]$
 - $[0, 24]$
 - $[-24, 24]$
- Let $f(x) = a \sin x + c$, where a and c are real numbers and $a > 0$. Then $f(x) < 0$, $\forall x \in R$ if
 - $c < -a$
 - $c > -a$
 - $-a < c < a$
 - $c < a$
- Sum of maximum and minimum value of the function $f(x) = \sin^2 x + 8 \cos x - 7$ is
 - 4
 - 5
 - 4
 - 5
- $\theta_1, \theta_2, \theta_3$ are angles of 1st quadrant if $\tan \theta_1 = \cos \theta_1$, $\tan \theta_2 = \operatorname{cosec} \theta_2$, $\cos \theta_3 = \theta_3$. Which of the following is not true?
 - $\theta_1 < \theta_2$
 - $\theta_1 < \theta_3$
 - $\theta_2 < \theta_3$
 - $\theta_3 < \theta_2$

Answers Key

Single Correct Answer Type

1. (c) 2. (d) 3. (d) 4. (b) 5. (c)

6. (d) 7. (c) 8. (c) 9. (b) 10. (a)
11. (b) 12. (c)

DPP 2.3

Trigonometric Ratios of Allied Angles

Single Correct Answer Type

- The value of $2 \cos 10^\circ + \sin 100^\circ + \sin 1000^\circ + \sin 10000^\circ$ is
(a) 0 (b) $\sin 10^\circ$ (c) $\cos 10^\circ$ (d) -1
- The two legs of a right triangle are $\sin \theta + \sin \left(\frac{3\pi}{2} - \theta \right)$ and $\cos \theta - \cos \left(\frac{3\pi}{2} - \theta \right)$. The length of its hypotenuse is
(a) 1 (b) 2
(c) $\sqrt{2}$ (d) none of these
- In cyclic quadrilateral $ABCD$ (none of these being 90°), which of the following is not true?
(a) $\tan A \cot C = -1$ (b) $\sec B \cos D = -1$
(c) $\operatorname{cosec} B \sin D = 1$ (d) None of these
- If $x = \sin 130^\circ \cos 80^\circ$, $y = \sin 80^\circ \cos 130^\circ$, $z = 1 + xy$, which one of the following is true?
(a) $x > 0, y > 0, z > 0$ (b) $x > 0, y < 0, 0 < z < 1$
(c) $x > 0, y < 0, z > 1$ (d) $x < 0, y < 0, 0 < z < 1$
- Suppose A and B are two angles such that $A, B \in (0, \pi)$, and satisfy $\sin A + \sin B = 1$ and $\cos A + \cos B = 0$. Then the possible value of $12 \cos 2A + 4 \cos 2B$ is equal to
(a) 4 (b) 6 (c) 8 (d) 12
- The sum $\sin \frac{\pi}{9} + \sin \frac{2\pi}{9} + \sin \frac{3\pi}{9} + \dots + \sin \frac{17\pi}{9}$ equals
(a) 0 (b) -1 (c) 1 (d) $-3/2$
- $\cos^2 73^\circ + \cos^2 47^\circ - \sin^2 43^\circ + \sin^2 107^\circ$ is equal to
(a) $1/2$ (b) $3/4$
(c) 1 (d) none of these
- The expression
$$\frac{\tan \left(x - \frac{\pi}{2} \right) \cdot \cos \left(\frac{3\pi}{2} + x \right) - \sin^3 \left(\frac{7\pi}{2} - x \right)}{\cos \left(x - \frac{\pi}{2} \right) \cdot \tan \left(\frac{3\pi}{2} + x \right)}$$
 simplifies to
(a) $(1 + \cos^2 x)$ (b) $\sin^2 x$
(c) $-(1 + \cos^2 x)$ (d) $\cos^2 x$
- The value of $\frac{\sin 300^\circ \cdot \tan 330^\circ \cdot \sec 420^\circ}{\tan 135^\circ \cdot \sin 210^\circ \cdot \sec 315^\circ}$ is
(a) -1 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$
- In triangle ABC , internal angle bisector $\angle A$ makes an angle θ with side BC . The value of $\sin \theta$ is equal to
(a) $\left| \sin \left(\frac{B-C}{2} \right) \right|$ (b) $\left| \sin \left(\frac{B}{2} - C \right) \right|$
(c) $\cos \left(\frac{B-C}{2} \right)$ (d) $\cos \left(\frac{B}{2} - C \right)$
- If the median of $\triangle ABC$ through A is perpendicular to AB , then
(a) $\tan A + \tan B = 0$ (b) $2 \tan A + \tan B = 0$
(c) $\tan A + 2 \tan B = 0$ (d) none of these
- The maximum value of $1 + \sin \left(\frac{\pi}{6} + \theta \right) + 2 \cos \left(\frac{\pi}{3} - \theta \right)$ for real values of θ is
(a) 3 (b) 5
(c) 4 (d) none of these

Answers Key

Single Correct Answer Type

- | | | | | | | | | | |
|--------|--------|--------|--------|--------|---------|---------|--------|--------|---------|
| 1. (c) | 2. (c) | 3. (d) | 4. (b) | 5. (c) | 6. (a) | 7. (c) | 8. (b) | 9. (c) | 10. (c) |
| | | | | | 11. (c) | 12. (c) | | | |

Trigonometric Ratios for Compound, Multiple, Sub-multiple Angles, and Transformation Formulas

DPP 3.1

Trigonometric Ratios of Compound Angles and Transformation Formulas

Single Correct Answer Type

- Given $a_1 \cos \alpha_1 + a_2 \cos \alpha_2 + \dots + a_n \cos \alpha_n = 0$ and $a_1 \cos (\alpha_1 + \theta) + a_2 \cos (\alpha_2 + \theta) + \dots + a_n \cos (\alpha_n + \theta) = 0$ ($\theta \neq k\pi$), then the value of $a_1 \cos (\alpha_1 + \lambda) + a_2 \cos (\alpha_2 + \lambda) + \dots + a_n \cos (\alpha_n + \lambda)$ is
(a) $\theta - \lambda$ (b) $\theta + \lambda$ (c) λ (d) 0
- $\frac{\cos^2 33^\circ - \cos^2 57^\circ}{\sin 21^\circ - \cos 21^\circ} =$
(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $-\frac{\sqrt{3}}{2}$
- The value of $\frac{2 \sin 40^\circ \cdot \sin 50^\circ \cdot \tan 10^\circ}{\cos 80^\circ}$ is
(a) $1/2$ (b) 1
(c) 2 (d) none of these
- The value of $\cos 65^\circ \cos 55^\circ \cos 5^\circ$ is
(a) $\frac{\sqrt{3}+1}{8\sqrt{2}}$ (b) $\frac{\sqrt{3}-1}{8\sqrt{2}}$
(c) $\frac{\sqrt{3}+1}{4\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{4\sqrt{2}}$
- If $\sin A + \sin 2A = x$ and $\cos A + \cos 2A = y$, then $(x^2 + y^2 - 3) =$
(a) $2y$ (b) y
(c) $3y$ (d) none of these
- If $\sin \theta + \sin 2\theta + \sin 3\theta = \sin \alpha$ and $\cos \theta + \cos 2\theta + \cos 3\theta = \cos \alpha$, then θ is equal to
(a) $\alpha/2$ (b) α (c) 2α (d) $\alpha/6$
- The value of $\frac{\sin 22^\circ \cos 8^\circ + \cos 158^\circ \cos 98^\circ}{\sin 23^\circ \cos 7^\circ + \cos 157^\circ \cos 97^\circ}$
(a) 1 (b) 2 (c) $1/2$ (d) $1/4$
- If $x = \cos \alpha + \cos \beta - \cos (\alpha + \beta)$ and $y = 4 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \left(\frac{\alpha + \beta}{2} \right)$, then $(x - y)$ equals
(a) 0 (b) 1 (c) -1 (d) -2

3.2 Trigonometry

9. The value of $\frac{\cos 5^\circ \cos 20^\circ + \cos 35^\circ \cos 50^\circ - \sin 5^\circ \sin 20^\circ - \sin 35^\circ \sin 50^\circ}{\sin 5^\circ \cos 20^\circ - \sin 35^\circ \cos 50^\circ + \cos 5^\circ \sin 20^\circ - \cos 35^\circ \sin 50^\circ}$ is
- (a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$
 (c) $-\sqrt{3}$ (d) $\sqrt{3}$
10. The value of $\sec 11^\circ \cdot \sec 19^\circ - 2 \cot 71^\circ$ is equal to
 (a) $2 \cot 11^\circ$ (b) $\tan 19^\circ$
 (c) $2 \tan 11^\circ$ (d) $\frac{1}{9} \tan 19^\circ$
11. α, β, γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$. The minimum value of the given expression $\sin \alpha + \sin \beta + \sin \gamma$ is
 (a) zero (b) -3
 (c) positive (d) negative
12. If $\sin \alpha = 1/\sqrt{5}$ and $\sin \beta = 3/5$, then $\beta - \alpha$ does not lie in the interval
 (a) $[0, \pi/4]$ (b) $[\pi/2, 3\pi/4]$
 (c) $[3\pi/4, \pi]$ (d) none of these
13. If in a triangle ABC , $\tan A + \tan B + \tan C$ has the value 6, then the value of $\cot A \cot B \cot C$ is
 (a) $1/6$ (b) 1
 (c) 6 (d) none of these
14. If $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$, then $\cos 2\alpha + \cos 2\beta =$
 (a) $-2 \sin (\alpha + \beta)$ (b) $-2 \cos (\alpha + \beta)$
 (c) $2 \sin (\alpha + \beta)$ (d) $2 \cos (\alpha + \beta)$
15. If α and β are acute such that $\alpha + \beta$ and $\alpha - \beta$ satisfy the equation $\tan^2 \theta - 4 \tan \theta + 1 = 0$, then $(\alpha, \beta) =$
 (a) $(30^\circ, 60^\circ)$ (b) $(45^\circ, 45^\circ)$
 (c) $(45^\circ, 30^\circ)$ (d) $(60^\circ, 45^\circ)$
16. Let α, β, γ be the measures of angle such that $\sin \alpha + \sin \beta + \sin \gamma \geq 2$. Then the possible value of $\cos \alpha + \cos \beta + \cos \gamma$ is
 (a) 3 (b) 2.5 (c) 2.4 (d) 2
17. The greatest value of $(2 \sin \theta + 3 \cos \theta + 4)^3 \cdot (6 - 2 \sin \theta - 3 \cos \theta)^2$, as $\theta \in R$, is
 (a) 2345 (b) 3456 (c) 1234 (d) 4567
18. Let $x, y \in R$ satisfy the condition such that $\sin x \sin y + 3 \cos y + 4 \sin y \cos x = \sqrt{26}$. The value of $\tan^2 x + \cot^2 y$ is equal to
 (a) 9×17 (b) 205
 (c) $\frac{1}{16} + \frac{9}{17}$ (d) none of these
19. If α, β, γ are acute angles and $\cos \theta = \sin \beta / \sin \alpha$, $\cos \phi = \sin \gamma / \sin \alpha$ and $\cos (\theta - \phi) = \sin \beta \sin \gamma$, then the value of $\tan^2 \alpha - \tan^2 \beta - \tan^2 \gamma$ is equal to
 (a) -1 (b) 0 (c) 1 (d) 2
20. In a $\triangle ABC$, If $\tan A/2, \tan B/2, \tan C/2$ are in A.P., then $\cos A, \cos B, \cos C$ are in
 (a) A.P. (b) G.P.
 (c) H.P. (d) none of these

Answers Key

Single Correct Answer Type

- | | | | | | | | | | |
|--------|--------|--------|--------|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (b) | 3. (b) | 4. (a) | 5. (a) | 11. (c) | 12. (b) | 13. (a) | 14. (b) | 15. (c) |
| 6. (a) | 7. (a) | 8. (b) | 9. (c) | 10. (c) | 16. (d) | 17. (b) | 18. (c) | 19. (b) | 20. (a) |

DPP 3.2

Trigonometric Ratios of Multiple Angles

Single Correct Answer Type

- The value of $\frac{\sin(\pi - \alpha)}{\sin \alpha - \cos \alpha \tan \frac{\alpha}{2}} - \cos \alpha$ is
(a) -2 (b) -1 (c) 1 (d) 2
- The expression $\frac{1 + \sin 2\alpha}{\cos(2\alpha - 2\pi) \cdot \tan(\alpha - \frac{3\pi}{4})} - \frac{1}{4} \sin 2\alpha$
 $\left[\cot \frac{\alpha}{2} + \cot \left(\frac{3\pi}{2} + \frac{\alpha}{2} \right) \right]$ is equal to
(a) 1 (b) 0
(c) $\sin^2(\alpha/2)$ (d) $\sin^2 \alpha$
- If $\tan \beta = \cos \theta \tan \alpha$, then $\tan^2 \frac{\theta}{2} =$
(a) $\frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)}$ (b) $\frac{\cos(\alpha - \beta)}{\cos(\alpha + \beta)}$
(c) $\frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)}$ (d) $\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)}$
- If $\operatorname{cosec} \theta = \frac{p+q}{p-q}$, then $\cot \left(\frac{\pi}{4} + \frac{\theta}{2} \right) =$
(a) $\sqrt{\frac{p}{q}}$ (b) $\sqrt{\frac{q}{p}}$
(c) $\sqrt{pq}$ (d) pq
- If $\sin(x + 3\alpha) = 3 \sin(\alpha - x)$, then
(a) $\tan x = \tan \alpha$ (b) $\tan x = \tan^2 \alpha$
(c) $\tan x = \tan^3 \alpha$ (d) $\tan x = 3 \tan \alpha$
- The value of $2(\cos^2 20^\circ + \cos^2 140^\circ + \cos^2 100^\circ)$ is
(a) 3/2 (b) 3
(c) 4 (d) none of these
- If $\sin 2\theta + \sin 2\phi = 1/2$ and $\cos 2\theta + \cos 2\phi = 3/2$, then $\cos^2(\theta - \phi) =$
(a) 3/8 (b) 5/8
(c) 3/4 (d) 5/4
- Let $f(x) = \sin^2(x + \alpha) + \sin^2(x + \beta) - 2 \cos(\alpha - \beta) \sin(x + \alpha) \sin(x + \beta)$. Which of the following is TRUE?
(a) $f(x)$ is strictly increasing in $x \in (\alpha, \beta)$
(b) $f(x)$ is strictly decreasing in $x \in (\alpha, \beta)$
(c) $f(x)$ is strictly increasing in $x \in \left(\alpha, \frac{\alpha + \beta}{2} \right)$ and strictly decreasing in $x \in \left(\frac{\alpha + \beta}{2}, \beta \right)$
(d) $f(x)$ is a constant function
- If A and B are positive acute angles satisfying the equalities $3 \cos^2 A + 2 \cos^2 B = 4$ and $\frac{3 \sin A}{\sin B} = \frac{2 \cos B}{\cos A}$, then $A + 2B$ is equal to
(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi/2$
- The value of the expression $\frac{\sin 40^\circ}{\sin 80^\circ} + \frac{\sin 80^\circ}{\sin 20^\circ} - \frac{\sin 20^\circ}{\sin 40^\circ}$ is
(a) 1 (b) 2 (c) 3 (d) 4
- If $\alpha = \frac{2\pi}{7}$, then $\tan \alpha \cdot \tan 2\alpha + \tan 2\alpha \cdot \tan 4\alpha + \tan 4\alpha \cdot \tan \alpha$ is
(a) -5 (b) -3 (c) -1 (d) -7
- The value of x satisfying the equation $x = \sqrt{2 + \sqrt{2 - \sqrt{2 + x}}}$ is
(a) $2 \cos 10^\circ$ (b) $2 \cos 20^\circ$
(c) $2 \cos 40^\circ$ (d) $2 \cos 80^\circ$
- If $3(\cos 2\phi - \cos 2\theta) = 1 - \cos 2\phi \cos 2\theta$, then $\tan \theta = k \tan \phi$, where $\theta, \phi \in \left(0, \frac{\pi}{2} \right)$, where $k =$
(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$
- A function $f(\theta) = \sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta$ is defined $\forall \theta \in \mathbb{R}$. Another function $g(\theta) = f\left(\frac{\pi}{2} - \theta\right)$ at the point θ where $f(\theta)$ is minimum, then the value of $\frac{1}{g(\theta)}$ is
(a) 2 (b) 5 (c) 3 (d) 4
- The number of integers in the range of $3 \sin^2 x + 3 \sin x \cdot \cos x + 7 \cos^2 x$ is
(a) 3 (b) 4 (c) 5 (d) 6
- The value of $\sum_{r=1}^{11} \tan^2 \left(\frac{r\pi}{24} \right)$ is
(a) 91 (b) 85
(c) 253/3 (d) none of these

3.4 Trigonometry

17. If $\theta_1, \theta_2, \theta_3$ are three values lying in $[0, 3\pi)$ for which $\tan \theta = \lambda$, then the value of

$$\left| \tan\left(\frac{\theta_1}{3}\right) \tan\left(\frac{\theta_2}{3}\right) + \tan\left(\frac{\theta_2}{3}\right) \tan\left(\frac{\theta_3}{3}\right) + \tan\left(\frac{\theta_3}{3}\right) \tan\left(\frac{\theta_1}{3}\right) \right|$$

is

- (a) $1/3$ (b) 1
(c) 3 (d) none of these
18. If $u = (1 + \cos \theta)(1 + \cos 2\theta) - \sin \theta \cdot \sin 2\theta$, $v = \sin \theta (1 + \cos 2\theta) + \sin 2\theta (1 + \cos \theta)$, then $u^2 + v^2 =$
- (a) $4(1 + \cos \theta)(1 + \cos 2\theta)$
(b) $4(1 + \sin \theta)(1 + \sin 2\theta)$
(c) $4(1 - \cos \theta)(1 - \cos 2\theta)$
(d) $4(1 - \sin \theta)(1 - \sin 2\theta)$

Multiple Correct Answers Type

19. If $\sec x + \tan x = \frac{22}{7}$, then

- (a) the value of $\tan \frac{x}{2} = \frac{29}{15}$
(b) the value of $\tan \frac{x}{2} = \frac{15}{29}$
(c) the value of $\operatorname{cosec} x + \cot x = \frac{29}{15}$
(d) the value of $\operatorname{cosec} x + \cot x = \frac{15}{29}$

20. If $y = \frac{\sqrt{1 - \sin 4x} + 1}{\sqrt{1 + \sin 4x} - 1}$, then y can be

- (a) $\cot x$ (b) $-\tan x$
(c) $-\cot\left(\frac{\pi}{4} + x\right)$ (d) $\tan\left(\frac{\pi}{4} + x\right)$

21. If $(a - b) \sin(\theta + \phi) = (a + b) \sin(\theta - \phi)$ and $a \tan \frac{\theta}{2} - b \tan \frac{\phi}{2} = c$, then

- (a) $b \tan \phi = a \tan \theta$ (b) $a \tan \phi = b \tan \theta$
(c) $\sin \phi = \frac{2bc}{a^2 - b^2 - c^2}$ (d) $\sin \theta = \frac{2ac}{a^2 - b^2 + c^2}$

22. If $\frac{\cos x - \cos \alpha}{\cos x + \cos \beta} = \frac{\sin^2 \alpha \cos \beta}{\sin^2 \beta \cos \alpha}$, then

- (a) $\cos x = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$
(b) $\cos x = \frac{\cos \alpha + \cos \beta}{1 - \cos \alpha \cos \beta}$

- (c) $\tan \frac{x}{2} = \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$
(d) $\tan \frac{x}{2} = -\tan \frac{\alpha}{2} \tan \frac{\beta}{2}$

23. If $0 \leq \theta \leq \pi$ and $\sin \frac{\theta}{2} = \sqrt{1 + \sin \theta} - \sqrt{1 - \sin \theta}$, then possible values of $\tan \theta$ is

- (a) $\frac{4}{3}$ (b) 0 (c) $\frac{-3}{4}$ (d) $\frac{-4}{3}$

24. For the expression $(\alpha \tan \gamma + \beta \cot \gamma)(\alpha \cot \gamma + \beta \tan \gamma) - 4\alpha\beta \cot^2 2\gamma$, which of the following is/are true?
- (a) dependent on α (b) independent of γ
(c) dependent on β (d) none of these

Answers Key

Single Correct Answer Type

1. (c) 2. (d) 3. (c) 4. (b) 5. (c)
6. (b) 7. (b) 8. (d) 9. (d) 10. (c)
11. (d) 12. (c) 13. (a) 14. (a) 15. (c)
16. (c) 17. (c) 18. (a)

Multiple Correct Answers Type

19. (b, c) 20. (a, b, c, d)
21. (b, c, d) 22. (a, c, d)
23. (b, d) 24. (a, b, c)

DPP 3.3

Trigonometric Ratios of Some Typical Angles, Conditional Identities and Miscellaneous

Single Correct Answer Type

- The value of $4 \cos 18^\circ - \frac{3}{\cos 18^\circ} - 2 \tan 18^\circ$ is equal to
(a) 0 (b) 1 (c) $\sqrt{5} - 1$ (d) $\sqrt{5} + 1$
- The value of $\frac{\cot 84^\circ \cot 48^\circ}{\cot 66^\circ \cot 78^\circ}$ is equal to
(a) 1 (b) 0
(c) $\sqrt{3}$ (d) none of these
- If $\tan \frac{\pi}{24} = (\sqrt{p} - \sqrt{q})(\sqrt{r} - s)$, then the value of $p + q + r + s$ is
(a) 6 (b) 7 (c) 8 (d) 9
- In a $\triangle ABC$, $2 \sin A \cos B + 2 \sin B \cos C + 2 \sin C \cos A = \sin A + \sin B + \sin C$, then $\triangle ABC$ is
(a) isosceles (b) right angled
(c) acute angled (d) none of these
- If $\cos A$, $\cos B$ and $\cos C$ are the roots of the cubic $x^3 + ax^2 + bx + c = 0$, where A, B, C are the angles of a triangle, then the value of $a^2 - 2b - 2c$ is
(a) 0 (b) $1/2$ (c) 1 (d) 2
- If $A + B + C + D = 2\pi$, then $\cos A + \cos B + \cos C + \cos D =$
(a) $4 \cos \frac{A+B}{2} \cos \frac{B+C}{2} \cos \frac{C+A}{2}$
(b) $4 \sin \frac{A+B}{2} \sin \frac{B+C}{2} \sin \frac{C+A}{2}$
(c) $1 - 4 \sin \frac{A+B}{2} \sin \frac{B+C}{2} \sin \frac{C+A}{2}$
(d) $-1 - 4 \cos \frac{A+B}{2} \cos \frac{B+C}{2} \cos \frac{C+A}{2}$
- If A, B, C, D are the angles of a quadrilateral, then $\frac{\tan A + \tan B + \tan C + \tan D}{\cot A + \cot B + \cot C + \cot D} =$
(a) $\cot A \cot B \cot C \cot D$
(b) $\tan A \tan B \tan C \tan D$
(c) $-\tan A \tan B \tan C \tan D$
(d) $-\cot A \cot B \cot C \cot D$
- The value of $\cos^2 \frac{\pi}{11} + \cos^2 \frac{2\pi}{11} + \cos^2 \frac{3\pi}{11} + \cos^2 \frac{4\pi}{11} + \cos^2 \frac{5\pi}{11}$ is
(a) $7/4$ (b) $9/4$ (c) $11/4$ (d) $13/4$
- The value of $\cos(-89^\circ) + \cos(-87^\circ) + \cos(-85^\circ) + \dots + \cos(85^\circ) + \cos(87^\circ) + \cos(89^\circ)$ is equal to
(a) $\operatorname{cosec} 1^\circ$ (b) $\sec 1^\circ$
(c) $2 \sec 1^\circ$ (d) $2 \operatorname{cosec} 1^\circ$
- If in a triangle ABC , $\cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2} = 4$, then the triangle is
(a) acute angled (b) obtuse angled
(c) right angled (d) none of these
- Consider the quantities such that $x_1, x_2, \dots, x_{10}, -1 \leq x_1, x_2, \dots, x_{10} \leq 1$ and $x_1^3 + x_2^3 + \dots + x_{10}^3 = 0$, then the maximum value of $x_1 + x_2 + \dots + x_{10}$ is
(a) $10/3$ (b) 10 (c) $5/3$ (d) 5
- The value of $\frac{\cos 25^\circ}{\sin 70^\circ \sin 85^\circ} + \frac{\cos 70^\circ}{\sin 25^\circ \sin 85^\circ} + \frac{\cos 85^\circ}{\sin 25^\circ \sin 70^\circ}$ is
(a) $1/2$ (b) 1 (c) 2 (d) $3/2$

Answers Key

Single Correct Answer Type

6. (a) 7. (b) 8. (b) 9. (a) 10. (d)
11. (a) 12. (c)

1. (a) 2. (a) 3. (c) 4. (a) 5. (c)

Trigonometric Equations

DPP 4.1

Basic Problems Related to Trigonometric Equations

Single Correct Answer Type

- The number of solutions of the equation $3^{2\sec^2 x} + 1 = 10 \cdot 3^{\tan^2 x}$ in the interval $[0, 2\pi]$ is
(a) 8 (b) 6 (c) 4 (d) 2
- If the sum of the roots of the equation $\sin^2 \theta = k$, ($0 < k < 1$) lying in $[0, 2\pi]$ is equal to the sum of the angles of a n -sided regular polygon, then the value of n is
(a) 6 (b) 4 (c) 2 (d) none of these
- The number of distinct of real roots of the equation $\tan^2 2x + 2 \tan 2x \tan 3x - 1 = 0$ in the interval $\left[0, \frac{\pi}{2}\right]$ is
(a) 1 (b) 2 (c) 3 (d) 4
- The number of solutions of equation $\cos 2x - 3 \cos x + 1 = \frac{1}{(\cot 2x - \cot x) \cdot \sin(x - \pi)}$ in $[0, 4\pi]$ is
(a) 0 (b) 2 (c) 4 (d) 8
- The number of distinct real roots of the equation $\sqrt{\sin x} - \frac{1}{\sqrt{\sin x}} = \cos x$ (where $0 \leq x \leq 2\pi$) is
(a) 1 (b) 2 (c) 3 (d) more than 3
- The number of distinct real roots of the equation $\sin^3 x + \sin^2 x + \sin x - \sin x \sin 2x - \sin 2x - 2 \cos x = 0$ belonging to the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(a) 0 (b) 1 (c) 2 (d) 3
- The number of solutions of the equation $\sqrt{13 - 18 \tan x} = 6 \tan x - 3$, where $-2\pi < x < 2\pi$ is
(a) 0 (b) 2 (c) 4 (d) 8
- The number of solutions of equation $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin \theta & 1 \\ 1 & 1 & 1 + \cot \theta \end{vmatrix} = 0$ in $\theta \in [0, 2\pi]$ is equal to
(a) 2 (b) 3 (c) 4 (d) 5
- The number of solutions of the equation $|\cos x| = \cos x - 2 \sin x$ in $[0, 6\pi]$ is
(a) 3 (b) 5 (c) 7 (d) 9
- The number of solutions of the equation $\log_5 \tan \theta = \log_5 4 \cdot \log_4 (3 \sin \theta)$ in $[0, 8\pi]$ is
(a) 0 (b) 2 (c) 4 (d) none of these
- If $\log_{10}(\sin x) + \log_{10}(\tan y) + \log_{10} 2 = 0$ and $\cot y = 2\sqrt{3} \cos x$, where $x \in (0, 4\pi)$; $y \in (0, 2\pi)$, then the number of ordered pairs (x, y) satisfying the equations simultaneously is
(a) 0 (b) 2 (c) 4 (d) 8
- The number of solutions of $3^{\sin 2x + 2 \cos^2 x} + 3^{1 + 2 \sin x (\sin x - \cos x)} = 28$ in $[0, 2\pi]$ is
(a) 3 (b) 4 (c) 5 (d) 6
- If the equation $a \sin^3 x + (b - a) \sin^2 x + (c - b) \sin x - c = 0$ has exactly three distinct solutions in $[0, \pi]$, where $a + b + c = 0$, then which of the following is not the possible value of c/a ?

4.2 Trigonometry

- (a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{2}}{7}$ (d) $\frac{2}{7}$

14. The number of roots of the equation

$$\sin\left(2x + \frac{\pi}{18}\right) \cdot \cos\left(2x - \frac{\pi}{9}\right) = \frac{-1}{4} \text{ lying in } [0, 2\pi] \text{ is}$$

- (a) 2 (b) 4 (c) 6 (d) 8

15. The number of solutions satisfying the equations $\tan 4\theta = \cot 5\theta$ and $\sin 2\theta = \cos \theta$ in $[0, 2\pi]$ is

- (a) 2 (b) 3 (c) 4 (d) 1

16. If n_1 denotes the maximum number of roots of $\sin \theta = k_1$ in $[0, 2\pi]$ and n_2 denotes the maximum number of roots of $\cos \theta = k_2$ in $[0, 2\pi]$, then

- (a) $n_1 + n_2 = 5$ (b) $n_1 + n_2 = 4$
(c) $n_1 + n_2 = 6$ (d) $n_1 + n_2 = 3$

Answers Key

Single Correct Answer Type

1. (c) 2. (a) 3. (c) 4. (a) 5. (b)
6. (b) 7. (c) 8. (a) 9. (c) 10. (c)

11. (c) 12. (b) 13. (a) 14. (b) 15. (c)
16. (a)

DPP 4.2

Trigonometric Equations of the Form $f(x) = f(\alpha)$, Where $f(x)$ is Trigonometric Function

Subjective Type

- Solve: If $\frac{1}{\cos \theta} + \frac{1}{\cos 3\theta} = \frac{1}{\cos 5\theta}$
- Solve: $3 - 2 \cos \theta - 4 \sin \theta - \cos 2\theta + \sin 2\theta = 0$.
- Find the general solution of the trigonometric equation $3^{\left(\frac{1}{2} + \log_3(\cos x + \sin x)\right)} - 2^{\log_2(\cos x - \sin x)} = \sqrt{2}$.
- Solve: $\cos 3x \cdot \cos^3 x + \sin 3x \cdot \sin^3 x = 0$
- Solve: $4 \cos^2 x \sin x - 2 \sin^2 x = 3 \sin x$.
- Solve: $1 + 2 \operatorname{cosec} x = -\frac{\sec^2 x}{2}$
- Solve: $2 \sin \left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8 \sin 2x \cdot \cos^2 2x}$, $x \in (0, 2\pi)$
- Solve the following system of equations for x and y :
 $5^{(\operatorname{cosec}^2 x - 3 \sec^2 y)} = 1$ and $2^{(2 \operatorname{cosec} x + \sqrt{3} |\sec y|)} = 64$
- Solve: $2 + \tan x \cdot \cot \frac{x}{2} + \cot x \cdot \tan \frac{x}{2} = 0$

Single Correct Answer Type

- $\frac{\sin 3\theta}{2 \cos 2\theta + 1} = \frac{1}{2}$ if $(n \in \mathbb{Z})$
 (a) $\theta = 2n\pi + \frac{\pi}{6}$ (b) $\theta = 2n\pi - \frac{\pi}{6}$
 (c) $\theta = n\pi + (-1)^n \frac{\pi}{6}$ (d) $\theta = n\pi - \frac{\pi}{6}$
- The general solution of the equation $\frac{1 - \sin x + \dots + (-1)^n \sin^n x + \dots}{1 + \sin x + \dots + \sin^n x + \dots} = \frac{1 - \cos 2x}{1 + \cos 2x}$ is $(n \in \mathbb{Z})$
 (a) $(-1)^n (\pi/6) + n\pi$
 (b) $(-1)^n (\pi/3) + n\pi$
 (c) $(-1)^{n+1} (\pi/6) + n\pi$
 (d) $(-1)^{n-1} (\pi/3) + n\pi$, $(n \in \mathbb{I})$
- The general solution of the equation, $2 \cot \frac{\theta}{2} = (1 + \cot \theta)^2$ is $(n \in \mathbb{Z})$

- (a) $n\pi + (-1)^n \frac{\pi}{4}$ (b) $n\pi + (-1)^n \frac{\pi}{6}$
 (c) $n\pi + (-1)^n \frac{\pi}{3}$ (d) none of these

13. If $\cos 2\theta = (\sqrt{2} + 1) \left(\cos \theta - \frac{1}{\sqrt{2}} \right)$, then the general value of θ ($n \in \mathbb{Z}$)

- (a) $2n\pi \pm \frac{\pi}{6}$ (b) $n\pi + \frac{\pi}{12}$
 (c) $n\pi + \frac{7\pi}{36}$ (d) none of these

14. The number of solutions of the equation $16(\sin^5 x + \cos^5 x) = 11(\sin x + \cos x)$ in the interval $[0, 2\pi]$ is
 (a) 6 (b) 7 (c) 8 (d) 9

15. The sum of solutions of $\sin \pi x + \cos \pi x = 0$ in $[0, 100]$ is
 (a) 4375 (b) 4975 (c) 5000 (d) 5025

16. If the equation $4 \cos^2 x \sin x - 2 \sin^2 x = 3 \sin x$, then x is equal to ($n \in \mathbb{Z}$)

- (a) $n\pi + \left(\frac{3\pi}{10}\right)$ (b) $n\pi + (-1)^{n+1} \left(\frac{3\pi}{10}\right)$
 (c) $n\pi + \left(\frac{3\pi}{10}\right)$ (d) none of these

17. The number of solutions of the equation

$$\cos^2 \left(\frac{\pi}{3} \cos x - \frac{8\pi}{3} \right) = 1 \text{ in the interval } [0, 10\pi] \text{ is}$$

- (a) 1 (b) 3 (c) 5 (d) 7

Multiple Correct Answers Type

18. If $\alpha \in [-2\pi, 2\pi]$ and $\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2} = \sqrt{2}$ ($\cos 36^\circ - \sin 18^\circ$), then a value of α

- (a) $\frac{7\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $-\frac{5\pi}{6}$ (d) $-\frac{\pi}{6}$

19. If $\sin \left(\frac{6}{5}x \right) = 0$ and $\cos \left(\frac{x}{5} \right) = 0$, then ($n \in \mathbb{Z}$)

- (a) $x = (n-5)\pi$ (b) $x = 6(n-1)\pi$
 (c) $x = 5 \left(n - \frac{1}{2} \right) \pi$ (d) $x = 5 \left(n + \frac{1}{2} \right) \pi$

4.4 Trigonometry

20. The equation $x^3 - \frac{3}{4}x = -\frac{\sqrt{3}}{8}$ is satisfied by

- (a) $x = \cos\left(\frac{5\pi}{18}\right)$ (b) $x = \cos\left(\frac{7\pi}{18}\right)$
 (c) $x = \cos\left(\frac{23\pi}{18}\right)$ (d) $x = -\sin\left(\frac{7\pi}{9}\right)$

21. If $5 \sin x \cos y = 1$, $4 \tan x = \tan y$, then

- (a) $x = (m+n)\frac{\pi}{2} + \frac{\pi}{4} + (-1)^m \frac{1}{2} \sin^{-1}\left(-\frac{3}{5}\right)$; $m, n \in Z$
 (b) $y = (n-m)\frac{\pi}{2} + \frac{\pi}{4} + (-1)^{m+1} \frac{1}{2} \sin^{-1}\left(-\frac{3}{5}\right)$; $m, n \in Z$
 (c) $x = (m+n)\frac{\pi}{2} + \frac{\pi}{4} + (-1)^m \frac{1}{2} \sin^{-1}\left(\frac{3}{5}\right)$; $m, n \in Z$
 (d) $y = (n-m)\frac{\pi}{2} + \frac{\pi}{4} + (-1)^{m+1} \frac{1}{2} \sin^{-1}\left(\frac{3}{5}\right)$; $m, n \in Z$

Answers Key

Subjective Type

1. $\theta = (2n+1)\frac{\pi}{14}$, $n \in Z$
 2. $\theta = (4n+1)\frac{\pi}{2}$ or $\theta = 2n\pi$
 3. $x = 2n\pi + \frac{3\pi}{4}$, $2n\pi + \frac{\pi}{12}$, $n \in Z$
 4. $x = \frac{n\pi}{2} \pm \frac{\pi}{4}$
 5. $x = n\pi$ or $x = n\pi + (-1)^n \frac{\pi}{10}$ or $x = n\pi + (-1)^n \left(-\frac{3\pi}{10}\right)$
 6. $x = 2n\pi - \frac{\pi}{2}$, $n \in Z$

7. $x = \frac{\pi}{12}, \frac{17\pi}{12}$

8. $x = n\pi + (-1)^n \frac{\pi}{6}$, $n \in Z$ and $y = m\pi \pm \frac{\pi}{6}$, $m \in Z$

9. $x = 2n\pi \pm \frac{2\pi}{3}$, $n \in Z$

Single Correct Answer Type

10. (c) 11. (a) 12. (b) 13. (d) 14. (a)
 15. (d) 16. (b) 17. (c)

Multiple Correct Answers Type

18. (a, d) 19. (c, d) 20. (a, b)
 21. (a, b)

DPP 4.3

Trigonometric Equations of the Form $a \cos x + b \sin x = c$ and Based on Extreme Values of the Function

Single Correct Answer Type

- If $\alpha < \beta < \gamma$ and $\sin \gamma \cdot \cos \alpha = 1$, where $\alpha, \gamma \in [\pi, 2\pi]$, then the least integral value of $f(x) = |x - \alpha| + |x - \beta| + |x - \gamma|$ is
(a) 0 (b) 1 (c) 2 (d) 3
- The number of solutions of the equation $\frac{1}{5^2} + 5^{\frac{1}{2} + \log_5(\sin x)} = 15^{\frac{1}{2} + \log_5(\cos x)}$ for $x \in [0, 100\pi]$ is
(a) 50 (b) 100 (c) 200 (d) 400
- Given $x + \sin y = 2009$ and $x + 2009 \cos y = 2008$, where $y \in \left[0, \frac{\pi}{2}\right]$, then $[x + y]$ equals, where $[\cdot]$ represents the greatest integer function,
(a) 2008 (b) 2009 (c) 2100 (d) 2010
- The number of solutions of equation $\sin \frac{5x}{2} - \sin \frac{x}{2} = 2$ in $[0, 2\pi]$ is
(a) 0 (b) 1 (c) 2 (d) 3
- The most general solution of $2^{\sin x} + 2^{\cos x} = 2^{\frac{1}{1-\sqrt{2}}}$ is $(n \in \mathbb{Z})$
(a) $2n\pi + \frac{5\pi}{4}$ (b) $n\pi - \frac{\pi}{4}$
(c) $n\pi + (-1)^n \frac{\pi}{4}$ (d) $2n\pi + \frac{\pi}{4}$
- If the equation $k \sin x + \sqrt{k-2} \cos x + (\tan \alpha + \cot \alpha) = 0$, $0 < \alpha < \frac{\pi}{2}$, possesses real solution, then k belongs to
(a) $(-\infty, -3] \cup [2, \infty)$ (b) $[-3, 2]$
(c) $[0, 2)$ (d) R
- If the equation $x^2 + 12 + 3 \sin(a + bx) + 6x = 0$ has at least one real solution, where $a, b \in [0, 2\pi]$, then the value of $a - 3b$ is $(n \in \mathbb{Z})$
(a) $2n\pi$ (b) $(2n+1)\pi$
(c) $(4n-1)\frac{\pi}{2}$ (d) $(4n+1)\frac{\pi}{2}$

- The number of solutions of the equation $\sin x \cdot \sin 2x \cdot \sin 3x = 1$ in $[0, 2\pi]$ is
(a) 0 (b) 1 (c) 2 (d) 3
- The solution set of the system of equation $x + y = 2\pi/3$ and $\cos x + \cos y = 3/2$, where x and y are real, are
(a) $n\pi + (-1)^n \sin^{-1} \frac{3}{2} + \frac{\pi}{6}$
(b) $2n\pi \pm \cos^{-1} \frac{3}{2} - \frac{\pi}{6}$
(c) no solution
(d) none of these
- The number of distinct real roots of the equation $\sin \pi x = x^2 - x + \frac{5}{4}$ is
(a) 0 (b) 1 (c) 2 (d) 4
- The number of ordered 5-tuple (u, v, w, x, y) , where $u, v, w, x, y \in [1, 11]$ which satisfy the inequality $2^{\sin^2 u + 3 \cos^2 v} \cdot 3^{\sin^2 w + \cos^2 x} \cdot 5^{\cos^2 y} \geq 720$ is
(a) 216 (b) 246 (c) 432 (d) 423
- The values of p for which $\sqrt{p} \sin x - 2 \cos x = \sqrt{2} + \sqrt{2-p}$ has solutions are
(a) $p > 0$ (b) $p \leq 3$
(c) $1 \leq p \leq 2$ (d) $\sqrt{5} - 1 \leq p \leq 2$

Multiple Correct Answers Type

- Which of the following are the solutions of equation $2 \sin 11x + \cos 3x + \sqrt{3} \sin 3x = 0$?
(a) $x = \frac{n\pi}{7} - \frac{\pi}{84}, n \in \mathbb{Z}$ (b) $x = \frac{n\pi}{4} + \frac{7\pi}{48}, n \in \mathbb{Z}$
(c) $x = \frac{n\pi}{7} - \frac{\pi}{63}, n \in \mathbb{Z}$ (d) $x = \frac{n\pi}{4} + \frac{\pi}{24}, n \in \mathbb{Z}$

Answers Key

Single Correct Answer Type

- | | | | | |
|---------|---------|--------|--------|---------|
| 1. (c) | 2. (a) | 3. (b) | 4. (a) | 5. (a) |
| 6. (a) | 7. (c) | 8. (a) | 9. (c) | 10. (b) |
| 11. (c) | 12. (d) | | | |

Multiple Correct Answers Type

13. (a, b)

DPP 4.4

Inequalities and Solving Equations Using Graphs

Single Correct Answer Type

- If $|\operatorname{cosec} x| = \frac{5\pi}{4} - \left| \frac{x}{2} \right| \forall x \in (-2\pi, 2\pi)$, then the number of solutions are
(a) 8 (b) 6 (c) 4 (d) 2
- The number of solutions of the equation $|\sin x| = |\cos 3x|$ in $[-2\pi, 2\pi]$ is
(a) 32 (b) 28 (c) 24 (d) 30
- If $6|\sin x| = x$ when $x \in [0, 2\pi]$, then the number of solutions are
(a) 0 (b) 3 (c) 5 (d) 4
- The number of solutions of equation $|\tan 2x| = \sin x$ in $[0, \pi]$
(a) 2 (b) 4 (c) 6 (d) 8
- The number of distinct real roots of the equation $x = \left(\frac{5\pi}{2} \right)^{\cos x}$
(a) 0 (b) 1 (c) 2 (d) 3
- If $\frac{\sin 3\theta}{\cos 2\theta} < 0$, then θ lies in
(a) $\left(\frac{3\pi}{8}, \frac{23\pi}{48} \right)$ (b) $\left(\frac{7\pi}{24}, \frac{3\pi}{8} \right)$
(c) $\left(\frac{13\pi}{48}, \frac{7\pi}{24} \right)$ (d) $\left(\frac{2\pi}{4}, \frac{7\pi}{12} \right)$
- If $2 \sin^2 \left(x - \frac{\pi}{3} \right) - 5 \sin \left(x - \frac{\pi}{3} \right) + 2 < 0$, then x belongs to
(a) $\left(\frac{(12n-5)\pi}{6}, \frac{(4n+1)\pi}{2} \right), n \in \mathbb{Z}$
(b) $\left(\frac{(6n-7)\pi}{6}, \frac{(2n+1)\pi}{2} \right), n \in \mathbb{Z}$

- (c) $\left(\frac{(4n+1)\pi}{6}, n\pi \right), n \in \mathbb{Z}$
(d) $\left((4n+1)\frac{\pi}{2}, (12n+7)\frac{\pi}{6} \right), n \in \mathbb{Z}$

- If equation $x^2 + 2x + 2 + e^\alpha - 2 \sin \beta = 0$ has a real solution, then $(n \in \mathbb{Z})$
(a) $\alpha, \beta \in \mathbb{R}$
(b) $\alpha \in (0, 1), \beta \in (\pi/6, \pi/2)$
(c) $\alpha \in (-\infty, 0], \beta \in (2n\pi + \pi/6, 2n\pi + 5\pi/6)$
(d) $\alpha \in (-\infty, 0], \beta \in (2n\pi - \pi/6)$

Multiple Correct Answers Type

- The function $f(x) = \sqrt{3} \sin x - \cos x$ will increase monotonically in the interval(s)
(a) $0 \leq x \leq \frac{\pi}{2}$ (b) $-\frac{\pi}{3} \leq x \leq \frac{2\pi}{3}$
(c) $\frac{5\pi}{3} \leq x \leq \frac{8\pi}{3}$ (d) $0 \leq x \leq \pi$
- If $\cos 3\theta + \sin 3\theta + (2 \sin 2\theta - 3)(\sin \theta - \cos \theta) > 0$, then θ lies in
(a) $\left(2n\pi - \frac{3\pi}{4}, 2n\pi + \frac{\pi}{4} \right), n \in \mathbb{Z}$
(b) $\left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{6} \right), n \in \mathbb{Z}$
(c) $\left(2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{3} \right), n \in \mathbb{Z}$
(d) $\left(2n\pi - \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4} \right), n \in \mathbb{Z}$

Answers Key

Single Correct Answer Type

1. (a) 2. (c) 3. (d) 4. (b) 5. (d)
6. (c) 7. (d) 8. (c)

Multiple Correct Answers Type

9. (b, c) 10. (a, b)

Inverse Trigonometric Functions

DPP 5.1

Inverse Trigonometric Functions: Domain, Range, Principal Values

Single Correct Answer Type

- The values of x which satisfy $18(\sin^{-1} x)^2 - 9\pi \sin^{-1} x + \pi^2 < 0$ and $18(\tan^{-1} x)^2 - 9\pi \tan^{-1} x + \pi^2 < 0$ simultaneously are
 - $\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{2}\right)$
 - $\left(\frac{\sqrt{3}}{2}, 1\right)$
 - $\left(\frac{1}{3}, \frac{\sqrt{3}}{2}\right)$
 - $\left(\frac{1}{\sqrt{3}}, \sqrt{3}\right)$
- If $f(x) = \sin^{-1}(\operatorname{cosec}(\sin^{-1} x)) + \cos^{-1}(\sec(\cos^{-1} x))$, then $f(x)$ takes
 - exactly two values
 - one value
 - undefined
 - infinite values
- The set of all real values of x satisfying $\sin^{-1} \sqrt{x} < \frac{\pi}{4}$, is
 - $\left(0, \frac{1}{2}\right)$
 - $\left[0, \frac{1}{2}\right)$
 - $\left(0, \frac{1}{2}\right]$
 - $\left[0, \frac{1}{2}\right]$
- The number of ordered triplets (x, y, z) that satisfy the equation $(\sin^{-1} x)^2 = \frac{\pi^2}{4} + (\sec^{-1} y)^2 + (\tan^{-1} z)^2$ is
 - 2
 - 4
 - 6
 - 8
- The range of function $f(x) = \sin^{-1}(x - \sqrt{x})$ is equal to
 - $\left[\sin^{-1} \frac{1}{4}, \frac{\pi}{2}\right]$
 - $\left[\sin^{-1} \frac{1}{2}, \frac{\pi}{2}\right]$
 - $\left[-\sin^{-1} \frac{1}{4}, \frac{\pi}{2}\right]$
 - $\left[-\sin^{-1} \frac{1}{2}, \frac{\pi}{2}\right]$
- The number of solutions of the equation $|\tan^{-1} x| = \sqrt{(x^2 + 1)^2 - 4x^2}$ is
 - 2
 - 3
 - 4
 - none of these
- The number of solutions of the equation $\sin^{-1} |x| = |\cos^{-1} x|$ are
 - 0
 - 1
 - 2
 - 3
- For $x \in (0, 1)$, let $\alpha = \sin^{-1} x$, $\beta = x$, $\gamma = \tan^{-1} x$, $\delta = \cot^{-1} x - \frac{\pi}{2}$. Which of the following is true?
 - $\alpha > \beta > \gamma$
 - $\beta > \alpha > \gamma > \delta$
 - $\alpha > \beta > \gamma > \delta$
 - $\beta > \alpha > \delta > \gamma$
- If $x, y, z \in R$ are such that they satisfy $x + y + z = 1$ and $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{4}$, then the value of $1x^3 + y^3 + z^3 - 31$ is
 - 1.5
 - 2
 - 2.5
 - 3
- The complete set of values of a for which the function $f(x) = \tan^{-1}(x^2 - 18x + a) > 0 \forall x \in R$ is
 - $(81, \infty)$
 - $[81, \infty)$
 - $(-\infty, 81)$
 - $(-\infty, 81]$

5.2 Trigonometry

11. The value of $\cos^{-1}\left(-\sin\frac{7\pi}{6}\right)$ is
- (a) $\frac{5\pi}{3}$ (b) $\frac{7\pi}{6}$
 (c) $\frac{\pi}{3}$ (d) none of these
12. The value of $\sec\left[\sin^{-1}\left(-\sin\frac{50\pi}{9}\right) + \cos^{-1}\cos\left(-\frac{31\pi}{9}\right)\right]$ is equal to
- (a) $\sec\frac{10\pi}{9}$ (b) $\sec\frac{\pi}{9}$
 (c) 1 (d) -1
13. The maximum value of the function $f(x) = (\sin^{-1}(\sin x))^2 - \sin^{-1}(\sin x)$ is
- (a) $\frac{\pi}{4}[\pi + 2]$ (b) $\frac{\pi}{4}[\pi - 2]$
 (c) $\frac{\pi}{2}[\pi + 2]$ (d) $\frac{\pi}{2}[\pi - 2]$
14. The solution of $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ is
- (a) $n\pi \pm 1, n\pi, n \in \mathbb{Z}$ (b) $n\pi + 1, n\pi, n \in \mathbb{Z}$
 (c) $n\pi - 1, n\pi, n \in \mathbb{Z}$ (d) $2n\pi + 1, n\pi, n \in \mathbb{Z}$
15. The value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is
- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$
 (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{5}$

Multiple Correct Answers Type

16. Let $f(x) = \cos^{-1}\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)$. Then which of the following

statement is/are true?

- (a) Range of $f(x)$ is $[0, \pi]$
 (b) $f(x) = \pi$ has infinite roots
 (c) $y = f(x)$ is identical with $y = \cos^{-1}(\cos x)$
 (d) $y = f(x)$ has period 2π
17. If $f(x) = \sin^{-1}(\sin x)$, $g(x) = \cos^{-1}(\cos x)$ and $h(x) = \cot^{-1}(\cot x)$, then which of the following is/are correct?
- (a) $f(x) = g(x) = h(x) \forall x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$
 (b) $f(x) < g(x) < h(x) \forall x \in \left(\frac{\pi}{2}, \pi\right)$
 (c) $h(x) > g(x) > f(x) \forall x \in \left(\frac{3\pi}{2}, 2\pi\right)$
 (d) $f(x) > g(x)$ has no real solution

Comprehension Type

For Q. 18–20

Let $f(x) = \sin^{-1}x + |\sin^{-1}x| + \sin^{-1}|x|$

18. The number of solution(s) of the equation $f(x) = x$ is

(a) 1 (b) 0 (c) 2 (d) 3

19. The range of $f(x)$ is

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[0, \frac{3\pi}{2}\right]$
 (c) $\left[0, \frac{\pi}{4}\right]$ (d) $[0, \pi]$

20. If the equation $f(x) = k$ has two solutions, then true set of values of k is

(a) $k \in \left(0, \frac{\pi}{2}\right)$ (b) $k \in \left[0, \frac{\pi}{2}\right]$
 (c) $k \in \left(0, \frac{\pi}{2}\right]$ (d) $k \in \left[0, \frac{\pi}{2}\right)$

Answers Key

Single Correct Answer Type

1. (a) 2. (b) 3. (b) 4. (a) 5. (c)
 6. (c) 7. (b) 8. (c) 9. (b) 10. (a)
 11. (c) 12. (d) 13. (a) 14. (a) 15. (c)

Multiple Correct Answers Type

16. (a, d) 17. (a, c, d)

Comprehension Type

18. (a) 19. (b) 20. (c)

DPP 5.2

Relating Different Inverse Trigonometric Functions, Complementary Angles

Single Correct Answer Type

1. Which of the following is not true?

- (a) $\sin \cos^{-1} \tan \cot^{-1} x = \sqrt{1 - \frac{1}{x^2}}$
 (b) $\cos \tan^{-1} \cot \sin^{-1} x = x$
 (c) $\tan \cot^{-1} \sin \cos^{-1} x = \frac{1}{\sqrt{1-x^2}}$
 (d) $\cot \sin^{-1} \cos \tan^{-1} x = \sqrt{1-x^2}$

2. The algebraic expression for

$$f(x) = \tan \left(\sin^{-1} \left(\cos \left(\tan^{-1} \frac{x}{2} \right) \right) \right) \text{ is}$$

- (a) $\frac{2}{x}$ (b) $\frac{x}{2}$ (c) $\frac{1}{x}$ (d) $\frac{2}{|x|}$

3. The value of x satisfying the equation $\cos^{-1} 3x + \sin^{-1} 2x = \pi$ is

- (a) $x = \frac{1}{\sqrt{3}}$ (b) $x = \frac{-1}{\sqrt{3}}$
 (c) $x = \frac{-1}{\sqrt{13}}$ (d) none of these

4. The minimum integral value of α for which the quadratic equation $(\cot^{-1} \alpha)x^2 - (\tan^{-1} \alpha)^{3/2}x + 2(\cot^{-1} \alpha)^2 = 0$ has both positive roots

- (a) 1 (b) 2 (c) 3 (d) 4

5. The number of roots of the equation

$$\sin^{-1} x - \frac{1}{\sin^{-1} x} = \cos^{-1} x - \frac{1}{\cos^{-1} x} \text{ is}$$

- (a) 0 (b) 1 (c) 2 (d) 3

6. The solution set of the inequality $\tan^{-1} x + \sin^{-1} x \geq \frac{\pi}{2}$ is

- (a) $[-1, 1]$ (b) $\left[\sqrt{\frac{5-1}{4}}, 1 \right]$
 (c) $\left[\sqrt{\frac{5-1}{2}}, 1 \right]$ (d) $\left[\frac{\sqrt{5-1}}{2}, 1 \right]$

7. The sum of all possible values of x satisfying the equation $\sin^{-1} (3x - 4x^3) + \cos^{-1} (4x^3 - 3x) = \frac{\pi}{2}$ is

- (a) -2 (b) -1 (c) 1 (d) 0

8. If maximum and minimum values of $|\sin^{-1} x| + |\cos^{-1} x|$ are M and m , then $M + m$ is

- (a) $\pi/2$ (b) π
 (c) 2π (d) 3π

9. If the function $f(x) = \sin^{-1} x + \cos^{-1} x$ and $g(x)$ are identical, then $g(x)$ can be equal to

- (a) $\sin^{-1} |x| + |\cos^{-1} x|$ (b) $\tan^{-1} x + \cot^{-1} x$
 (c) $|\sin^{-1} x| + |\cos^{-1} x|$ (d) $(\sqrt{\sin^{-1} x})^2 + (\sqrt{\cos^{-1} x})^2$

10. The value of x satisfying $\sin^{-1} \left(\sqrt{\frac{3x-1}{25}} \right) + \sin^{-1} \left(\sqrt{\frac{3x+1}{25}} \right) = \frac{\pi}{2}$ lies in the interval

- (a) (1, 2) (b) (2, 3)
 (c) (3, 4) (d) (4, 5)

11. The set of values of k for which the equation $\sin^{-1} x + \cos^{-1} x + \pi(|x| - 2) = k\pi$ possesses real solution is $[a, b]$ then the value of $a + b$ is

- (a) 0 (b) -2 (c) -1 (d) 2

12. The solution set of inequality $(\sin x + \cos^{-1} x) - (\cos x - \sin^{-1} x) \geq \frac{\pi}{2}$ is equal to

- (a) $\left[\frac{\pi}{4}, \frac{5\pi}{4} \right]$ (b) $\bigcup_{n \in \mathbb{I}} \left[2n\pi + \frac{\pi}{4}, 2n\pi + \frac{5\pi}{4} \right]$
 (c) $\left[\frac{\pi}{4}, 1 \right]$ (d) $\left[-1, \frac{-\pi}{4} \right] \cup \left[\frac{\pi}{4}, 1 \right]$

13. The number of integral values in the range of the function $f(x) = \sin^{-1} x - \cot^{-1} x + x^2 + 2x + 6$ is

- (a) 10 (b) 11 (c) 12 (d) 8

Multiple Correct Answers Type

14. If $\sin^{-1} \left(\frac{\sqrt{x}}{2} \right) + \sin^{-1} \left(\sqrt{1 - \frac{x}{4}} \right) + \tan^{-1} y = \frac{2\pi}{3}$, then

- (a) maximum value of $x^2 + y^2$ is $\frac{49}{3}$

5.4 Trigonometry

- (b) maximum value of $x^2 + y^2$ is 4
- (c) minimum value of $x^2 + y^2$ is $\frac{1}{3}$
- (d) minimum value of $x^2 + y^2$ is 3

15. $\cos^{-1} \left(\sqrt{\frac{a-x}{a-b}} \right) = \sin^{-1} \left(\sqrt{\frac{x-b}{a-b}} \right)$ is possible, if

- (a) $a > x > b$
- (b) $a < x < b$
- (c) $a = x = b$
- (d) $a > b$ and x takes any value

Answers Key

Single Correct Answer Type

- 1. (d) 2. (d) 3. (d) 4. (b) 5. (c)
- 6. (c) 7. (d) 8. (c) 9. (c) 10. (d)
- 11. (b) 12. (c) 13. (d)

Multiple Correct Answers Type

- 14. (a, c) 15. (a, b)

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DPP 5.3

Sum and Difference of Angles

Single Correct Answer Type

- The sum of the roots of the equation $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1} \frac{3}{5}$ is
(a) -2 (b) -1 (c) 0 (d) 2
- If x and y are positive integer satisfying $\tan^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{1}{y}\right) = \frac{\pi}{7}$, then the number of ordered pairs of (x, y) is
(a) 3 (b) 4 (c) 5 (d) 6
- Number of roots of equation $\sin^{-1}(1-x) + 2\sin^{-1}x = \pi/2$, is
(a) 0 (b) 1 (c) 2 (d) 3
- If $x > 1$ and $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$, then $x =$
(a) 2 (b) $\sqrt{3}$ (c) 4 (d) 3
- The solution of $\sin^{-1}x - \sin^{-1}2x = \pm \frac{\pi}{3}$ is
(a) $\pm \frac{1}{3}$ (b) $\pm \frac{1}{4}$
(c) $\pm \frac{\sqrt{3}}{2}$ (d) $\pm \frac{1}{2}$
- If $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = p$ for all $x \in [-1, 0]$, then p is equal to
(a) $-\frac{\pi}{2}$ (b) 0
(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$
- Let $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $g(x) = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$. Then the value of $f(10) - g(100)$ is equal to
(a) $\pi - 2(\tan^{-1}(10) + \tan^{-1}(100))$
(b) 0
(c) $2(\tan^{-1}(100) - \tan^{-1}(10))$
(d) $2(\tan^{-1}(10) - \tan^{-1}(100))$
- The number of real solution of equation $\tan^{-1}x + \cot^{-1}(-|x|) = 2\tan^{-1}(6x)$ is
(a) 4 (b) 3 (c) 2 (d) 1
- The number of roots of the equation $\sin^{-1}\frac{3x}{5} + \sin^{-1}\frac{4x}{5} = \sin^{-1}x$ is
(a) 0 (b) 1 (c) 2 (d) 3
- If $x \in \left[-1, \frac{-1}{\sqrt{2}}\right]$, then the inverse of the function $f(x) = \sin^{-1}(2x\sqrt{1-x^2})$ is given by
(a) $-\cos \frac{y}{2}$ (b) $\cos \frac{y}{2}$
(c) $-2\cos y$ (d) $-2\cos y$
- The expression $\sum_{n=1}^{\infty} \cot^{-1}(n^2 - 3n + 3)$ simplifies to
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{4}$ (d) π
- The value of sum $\sum_{n=1}^{\infty} \cot^{-1}\left(\frac{(n^2+2n)(n^2+2n+1)+1}{2n+2}\right)$ is equal to
(a) $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ (b) $\sec^{-1}\left(\frac{\sqrt{5}}{2}\right)$
(c) $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ (d) $\cot^{-1}(1)$
- The number of solution of the equation $2\sin^{-1}\left(\frac{2x}{1+x^2}\right) - \pi x^3 = 0$ is equal to
(a) 0 (b) 1
(c) 2 (d) 3

Multiple Correct Answers Type

- The solutions of the equation $\sin[2\cos^{-1}\{\cot(2\tan^{-1}x)\}] = 0$ are
(a) ± 1 (b) $1 \pm \sqrt{2}$
(c) $-1 \pm \sqrt{2}$ (d) $\pm \sqrt{2}$

5.6 Trigonometry

15. Let x_1, x_2, x_3, x_4 be four non-zero numbers satisfying the equation $\tan^{-1} \frac{a}{x} + \tan^{-1} \frac{b}{x} + \tan^{-1} \frac{c}{x} + \tan^{-1} \frac{d}{x} = \frac{\pi}{2}$. Then

- (a) $x_1 + x_2 + x_3 + x_4 = a + b + c + d$
 (b) $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} = 0$
 (c) $x_1 x_2 x_3 x_4 = abcd$
 (d) $(x_2 + x_3 + x_4)(x_3 + x_4 + x_1)(x_4 + x_1 + x_2)(x_1 + x_2 + x_3) = abcd$

16. Which of the following is/are true?

- (a) $\tan^{-1} \frac{1}{3} = \frac{1}{2} \sin^{-1} \frac{3}{5}$
 (b) $\tan^{-1} \frac{1}{3} = \frac{\pi}{4} - \cot^{-1} 2$
 (c) $\tan^{-1} \frac{1}{3} = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{4}{5}$
 (d) $\tan^{-1} \frac{1}{3} = \frac{\pi}{2} - \cot^{-1} 3$

Answers Key

Single Correct Answer Type

1. (c) 2. (d) 3. (c) 4. (b) 5. (d)
 6. (b) 7. (c) 8. (c) 9. (d) 10. (a)
 11. (c) 12. (c) 13. (d)

Multiple Correct Answers Type

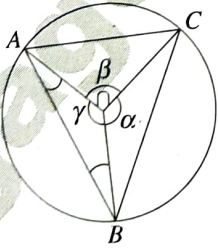
14. (a, b, c) 15. (b, c, d) 16. (a, b, c)

Solutions and Properties of Triangle

DPP 6.1

Different Rules in Triangle

Single Correct Answer Type

- In a triangle $\angle A = 55^\circ$, $\angle B = 15^\circ$, $\angle C = 110^\circ$. Then $c^2 - a^2$ is equal to
 (a) ab (b) $2ab$
 (c) $-ab$ (d) none of these
- In a triangle ABC if $2a = \sqrt{3}b + c$, then possible relation is
 (a) $c^2 = a^2 + b^2 - ab$ (b) $a^2 = b^2 + c^2$
 (c) $b^2 = a^2 + c^2 - ac\sqrt{3}$ (d) $a = b = c$
- A circle of area 20 sq. units is centered at the point O . Suppose $\triangle ABC$ is inscribed in that circle and has area 8 sq. units. The central angles α , β and γ are as shown in the figure. The value of $(\sin \alpha + \sin \beta + \sin \gamma)$ is equal to

 (a) $\frac{4\pi}{5}$ (b) $\frac{3\pi}{4}$ (c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{4}$
- In $\triangle ABC$, $bc = 2b^2 \cos A + 2c^2 \cos A - 4bc \cos^2 A$, then $\triangle ABC$ is
 (a) isosceles but not necessarily equilateral
 (b) equilateral
 (c) right angled but not necessarily isosceles
 (d) right angled isosceles
- Given a triangle $\triangle ABC$ such that $\sin^2 A + \sin^2 C = 1001 \times \sin^2 B$. Then the value of $\frac{2(\tan A + \tan C) \cdot \tan^2 B}{\tan A + \tan B + \tan C}$ is
 (a) $\frac{1}{2000}$ (b) $\frac{1}{1000}$ (c) $\frac{1}{500}$ (d) $\frac{1}{250}$
- $\triangle ABC$ has different side lengths a, b, c . If a^2, b^2, c^2 as sides form another $\triangle PQR$, then $\triangle ABC$ will always be
 (a) acute angled triangle only
 (b) obtuse angled triangle only
 (c) sometimes acute or sometimes obtuse depending on values of a, b and c
 (d) none of these
- In $\triangle ABC$, $\angle B = \frac{\pi}{4}$, $\angle C = \frac{\pi}{6}$. D is a point on BC which divides it in the ratio 1 : 3, $\angle DAB = \beta$, then
 (a) $\left(\sec \frac{\pi}{6}\right) AB + \cot \beta = \cot\left(\frac{\pi}{6}\right) AC + (4\sqrt{3} - 5)$
 (b) $\left(\sec \frac{\pi}{4}\right) AB \cot \beta = \cot\left(\frac{\pi}{4}\right) AC(4\sqrt{3} - 5)$
 (c) $\left(\sec \frac{\pi}{4}\right) AB + \cot \beta = \cot\left(\frac{\pi}{6}\right) AC + (4\sqrt{3} + 5)$
 (d) $\left(\sec \frac{\pi}{6}\right) AB \cot \beta = \cot\left(\frac{\pi}{4}\right) AC(\sqrt{3} + 5)$

8. The acute angle of a rhombus whose side is the geometric mean of its diagonals is
(a) 15° (b) 20° (c) 30° (d) 80°
9. Let sides a, b and c of $\triangle ABC$ be related by the relation $a : b : c = 3 : 5 : 4$. Altitudes AD, BE and CF are dropped on BC, CA and AB , respectively. Let mid points of BC, CA and AB be P_1, P_2 and P_3 , respectively. If $P_1D + P_2E + P_3F = 42$, then the value of $a + b + c$ is
(a) 1200 (b) 120 (c) 12 (d) none of these
10. Triangle ABC is right angled at A . The points P and Q are on hypotenuse BC such that $BP = PQ = QC$. If $AP = 3$ and $AQ = 4$, then length BC is equal to
(a) $3\sqrt{5}$ (b) $5\sqrt{3}$ (c) $4\sqrt{5}$ (d) 7
11. ABC is a right angled triangle of which A is the right angle, BD is drawn perpendicular to BC and meets CA produced in D . If $AB = 12, AC = 16, BC = 20$, then $BD =$
(a) 15 (b) 25 (c) 10 (d) 225
12. In a $\triangle ABC$, the median AD is perpendicular to AC . If $b = 5$ and $c = 11$, then $a =$
(a) 10 (b) 12 (c) 14 (d) $\sqrt{221}$
13. ABC is an equilateral triangle where $AB = a$ and P is any point in its plane such that $PA = PB + PC$. Then $\frac{PA^2 + PB^2 + PC^2}{a^2}$ is
(a) 3 (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{3}{4}$ (d) 2
15. If A, B, C are the angles of a triangle such that $\sin^2 A + \sin^2 B = \sin^2 C$, then
(a) $\sin A + \sin B > \sin C$
(b) $\tan A \tan B = 1$
(c) $\sin A + \sin B = 1$
(d) $\tan A \cdot \tan B < 1$
16. In $\triangle ABC, \angle C = 2\angle A$ and $AC = 2BC$. Then which of the following is/are True?
(a) Angles A, B, C are in arithmetic progression.
(b) Angles A, C, B are in arithmetic progression
(c) $\triangle ABC$ is a right angled isosceles triangle
(d) $BC^2 + CA^2 + AB^2 = 8R^2$, where R is the circum-radius of $\triangle ABC$
17. In $\triangle ABC, AB = 9, AC = 17.5$, altitude from A to line BC cut at $M, AM = 3$. Then
(a) radius of circle which circumscribe $\triangle ABC$ is 26.25
(b) radius of circle which circumscribe $\triangle ABM$ is 4.5
(c) orthocentre of $\triangle ABC$ lies outside $\triangle ABC$
(d) orthocentre of $\triangle ABC$ lies inside $\triangle ABC$
18. If in a triangle ABC, θ is the angle determined by $\cos \theta = \frac{(a-b)/c}{2}$, then
(a) $\frac{(a+b)\sin \theta}{2\sqrt{ab}} = \cos \frac{A-B}{2}$
(b) $\frac{(a+b)\sin \theta}{2\sqrt{ab}} = \cos \frac{A+B}{2}$
(c) $\frac{c \sin \theta}{2\sqrt{ab}} = \cos \frac{A-B}{2}$
(d) $\frac{c \sin \theta}{2\sqrt{ab}} = \cos \frac{A+B}{2}$

Multiple Correct Answers Type

14. Let a, b, c be the sides of a triangle $ABC, a = 2c, \cos(A-C) + \cos B = 1$. Then the value of C is
(a) $\pi/6$ (b) $\pi/3$ (c) $2\pi/3$ (d) $5\pi/6$

Answers Key

Single Correct Answer Type

1. (a) 2. (b) 3. (a) 4. (a) 5. (d)
6. (a) 7. (b) 8. (c) 9. (a) 10. (a)
11. (a) 12. (c) 13. (d)

Multiple Correct Answers Type

14. (a, d) 15. (a, b) 16. (b, d)
17. (a, b, c) 18. (a, d)

DPP 6.2

Half-Angle Formula and Area of Triangle

Single Correct Answer Type

- In $\triangle ABC$ if $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} = \frac{2}{3}$, then $a + c$
 - $3b$
 - $2b$
 - $3b/2$
 - $4b$
- In a triangle ABC if $\tan \frac{A}{2} \cdot \tan \frac{B}{2} = \frac{1}{3}$ and $ab = 4$, then the value of c can be
 - 1
 - 1.5
 - 2.5
 - none of these
- In a triangle ABC if $\cot \frac{A}{2} \cot \frac{B}{2} = c$, $\cot \frac{B}{2} \cot \frac{C}{2} = a$ and $\cot \frac{C}{2} \cot \frac{A}{2} = b$, then $\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} =$
 - 1
 - 0
 - 1
 - 2
- The perimeter of a triangle ABC right angled at C is 70, and the inradius is 6, then $|a - b| =$
 - 1
 - 2
 - 8
 - 9
- The diagonals of a parallelogram are inclined to each other at an angle of 45° , while its sides a and b ($a > b$) are inclined to each other at an angle of 30° . then the value of $\frac{a}{b}$ is
 - $\frac{3}{2}$
 - $\frac{3 + \sqrt{5}}{2}$
 - $\frac{3 + \sqrt{5}}{4}$
 - $\frac{\sqrt{5} + 1}{2}$
- In a triangle ABC if $2\Delta^2 = \frac{a^2 b^2 c^2}{a^2 + b^2 + c^2}$, then it is
 - equilateral
 - isosceles but not right angled
 - isosceles right angled
 - right angled
- If in any triangle, the area $\Delta \leq \frac{b^2 + c^2}{\lambda}$, then the largest possible numerical value of λ is
 - 1
 - 2
 - 3
 - 4
- In any triangle ABC , if $2\Delta a - b^2 c = c^3$, (where Δ is the area of triangle), then which of the following is possible?
 - B is obtuse
 - A is obtuse
 - C is obtuse
 - B is right angle
- In $\triangle ABC$, $a^2(s-a) + b^2(s-b) + c^2(s-c) =$
 - $4R(\cos A + \cos B + \cos C)$
 - $4R\Delta(\sin A + \sin B + \sin C)$
 - $4R\Delta \left(1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}\right)$
 - none of these
- Let ABC be an equilateral triangle and $KLMN$ be a rectangle with K, L on BC , M on AC and N on AB . Suppose $AN/NB = 2$ and the area of $\triangle BKN$ is 6. The area of the triangle ABC is
 - 54
 - 108
 - 48
 - none of these

Multiple Correct Answers Type

- Let a, b, c be the sides BC, CA, AB of $\triangle ABC$ on xy plane. If abscissa and ordinate of vertices of the triangle are integers and R is its circumradius, then $2R$ can be equal to
 - $\frac{8}{9} abc$
 - abc
 - $\frac{9}{8} abc$
 - $\frac{abc}{2}$
- In a $\triangle ABC$, if $\tan \frac{A}{2} = \frac{5}{6}$, $\tan \frac{B}{2} = \frac{20}{37}$, then which of the following is/are correct?
 - $\angle B > \angle C$
 - $\angle B < \angle C$
 - $a > b > c$
 - $a < b < c$
- If area of $\triangle ABC$ (Δ) and angle C are given and if the side c opposite to the given angle is minimum, then
 - $a = \sqrt{\frac{2\Delta}{\sin C}}$
 - $b = \sqrt{\frac{2\Delta}{\sin C}}$
 - $a = \frac{4\Delta}{\sin C}$
 - $b = \frac{4\Delta}{\sin^2 C}$

Answers Key

Single Correct Answer Type

- (b)
- (c)
- (d)
- (a)
- (d)
- (d)
- (d)
- (b)
- (c)
- (b)

Multiple Correct Answers Type

- (a, b, d)
- (a, c)
- (a, b)

DPP 6.3

Different Centres of Triangle, Angle and Length Associated with It

Single Correct Answer Type

1. Let ABC is an acute angled triangle with circumcenter O , orthocenter H . If $AO = AH$, then the measure of angle A is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

2. In a triangle ABC if $\angle ABC = 60^\circ$, then $\left(\frac{AB - BC + CA}{r}\right)^2 =$

(a) 10 (b) 11 (c) 12 (d) 14

3. The area of an acute triangle ABC is Δ , the area of its pedal triangle is ' p ', where $\cos B = \frac{2p}{\Delta}$ and $\sin B = \frac{2\sqrt{3}p}{\Delta}$. The

value of $8(\cos^2 A \cos B + \cos^2 C)$ is

(a) 1 (b) 2
(c) 3 (d) none of these

4. In a triangle ABC , D is a point on BC such that AD is the internal bisector of $\angle A$. Let $\angle B = 2\angle C$ and $CD = AB$. Then $\angle A$ is

(a) 18° (b) 36° (c) 54° (d) 72°

5. In ΔABC , circumradius is 3 and inradius is 1.5 units. The value of $a \cot^2 A + b^2 \cot^3 B + c^3 \cot^4 C$ is

(a) $13\sqrt{3}$ (b) $11\sqrt{6}$
(c) 21 (d) none of these

6. AD, BE, CF are internal angular bisectors of ΔABC and I

is the incentre. If $a(b+c)\sec \frac{A}{2} ID + b(a+c)\sec \frac{B}{2} IE +$

$c(a+b)\sec \frac{C}{2} IF = kabc$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

7. In ΔABC it is given that distance between the circumcentre (O) and orthocentre (H) is $R\sqrt{1 - 8 \cos A \cos B \cos C}$. If Q is the midpoint of OH , then AQ is

(a) $\frac{R}{2} \sqrt{1 + 8 \cos A \sin B \sin C}$

(b) $R \sqrt{1 + 8 \cos A \sin B \sin C}$

(c) $2R \sqrt{1 + 8 \cos A \sin B \sin C}$

(d) $\frac{R}{2} \sqrt{1 + 8 \sin A \cos B \cos C}$

8. In any ΔABC , line joining circumcentre (O) and incentre (I) is parallel to AC , then OI is equal to

(a) $R \left| \tan \left(\frac{A-C}{2} \right) \right|$ (b) $R \tan (A-C)$

(c) $R \left| \sec \left(\frac{A-C}{2} \right) \right|$ (d) $R \sec (A-C)$

9. Let the incircle of a ΔABC touches sides BC, CA and AB at D, E and F , respectively. Let area of ΔABC be Δ and that of ΔDEF be Δ' . If a, b and c are sides of ΔABC , then

the value of $abc(a+b+c) \frac{\Delta'}{\Delta^3}$ is

(a) 1 (b) 2 (c) 3 (d) 4

10. Let H be the orthocentre of triangle ABC . Then angle subtended by side BC at the centre of incircle of ΔCHB is

(a) $\frac{A}{2} + 90^\circ$

(b) $\frac{B+C}{2} + 90^\circ$

(c) $\frac{B-C}{2} + 90^\circ$

(d) None of these

Multiple Correct Answers Type

11. Let ' P ' be an interior point of ΔABC . If $\angle A = 45^\circ$, $\angle B = 60^\circ$ and $\angle C = 75^\circ$. If X = area of ΔPBC , Y = area of ΔPAC and Z = area of ΔPAB , then which of the following ratios is/are true?

(a) If P is the centroid, then $X : Y : Z$ is $1 : 1 : 1$

(b) If P is the incentre, then $X : Y : Z$ is $2 : \sqrt{6} : (\sqrt{3} + 1)$

(c) If P the orthocentre, then $X : Y : Z$ is $1 : \sqrt{3} : (2 + \sqrt{3})$

(d) If P is the circumcentre, then $X : Y : Z$ is $2 : \sqrt{3} : 1$

12. Let ' l ' is the length of median from the vertex A to the side BC of a ΔABC . Then

(a) $4l^2 = 2b^2 + 2c^2 - a^2$

(b) $4l^2 = b^2 + c^2 + 2bc \cos A$

(c) $4l^2 = a^2 + 4bc \cos A$

(d) $4l^2 = (2s - a)^2 - 4bc \sin^2 \frac{A}{2}$

13. A circle having centre as O' and radius r' touches the incircle of ΔABC externally at F , where F is on BC and

also touches its circumcircle internally at G . If O is the circumcentre of $\triangle ABC$ and I is its incentre, then

- (a) $OO' = R - r'$
 (b) Perpendicular distance from O to line joining IO' is $\left| \frac{b-c}{2} \right|$
 (c) Projection of OO' on line joining $IO' = r' + R \cos A$
 (d) $r' = \frac{\Delta}{a} \tan^2 A$

Comprehension Type

For Q. 14–16

Let ABC be a triangle in which the line joining the circumcentre and incentre is parallel to base BC of the triangle. Then answer the following questions:

14. Then range of $\angle A$ is

- (a) $\left[\frac{\pi}{6}, \frac{\pi}{3} \right]$ (b) $\left[\frac{\pi}{3}, \frac{\pi}{2} \right]$
 (c) $\left[\frac{\pi}{3}, \frac{2\pi}{3} \right] - \left\{ \frac{\pi}{2} \right\}$ (d) $\left[0, \frac{\pi}{2} \right]$

15. If $ODEI$ is a square where O and I stands for circumcentre and incentre, respectively and D and E are the point of perpendicular from O and I on the base BC , then

- (a) $\frac{r}{R} = \frac{3}{8}$ (b) $\frac{r}{R} = 2 - \sqrt{3}$
 (c) $\frac{r}{R} = \sqrt{2} - 1$ (d) $\frac{r}{R} = \frac{1}{4}$

16. If $\angle A = 60^\circ$, then $\triangle ABC$ is

- (a) isosceles (b) right angled
 (c) right angled isosceles (d) equilateral

For Q. 17–19

Incircle of $\triangle ABC$ touches AB, BC, CA at R, P, Q , respectively.

If $\frac{2}{AR} + \frac{5}{BP} + \frac{5}{CQ} = \frac{6}{r}$ and the perimeter of the triangle is the smallest integer, then answer the following questions:

17. $\triangle ABC$ is

- (a) scalene (b) isosceles
 (c) equilateral (d) right angled

18. The inradius of incircle of $\triangle ABC$ is

- (a) 4 (b) 3 (c) 2 (d) 1

19. The area of $\triangle ABC$ is

- (a) 15 sq. units (b) 21 sq. units
 (c) 24 sq. units (d) 27 sq. units

Answers Key

Single Correct Answer Type

1. (c) 2. (c) 3. (c) 4. (d) 5. (a)
 6. (b) 7. (a) 8. (a) 9. (d) 10. (b)

Multiple Correct Answers Type

11. (a, b, c, d) 12. (a, b, c, d)
 13. (a, b, c, d)

Comprehension Type

14. (b) 15. (c) 16. (d) 17. (b) 18. (c)
 19. (d)

DPP 6.4

Exradius, Ambiguous Cases and Miscellaneous

Single Correct Answer Type

- If in a triangle ABC , $r_1 + r_2 + r_3 = 9r$, then the triangle is necessarily
 (a) right angled (b) equilateral
 (c) obtuse angled (d) none of these
- In triangle ABC , $r = \frac{R}{6}$ and $r_1 = 7r$. Then the measure of angle $A =$
 (a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$
- $(r_2 + r_3) \sqrt{\frac{r_1}{r_2 r_3}} =$
 (a) a (b) b (c) c (d) bc
- In $\triangle ABC$, right angled at A , $\cos^{-1} \left(\frac{R}{r_2 + r_3} \right)$ is
 (a) 30° (b) 60° (c) 90° (d) 45°
- If in triangle ABC , $r_1 = 2r_2 = 3r_3$, D is the middle point of BC . Then $\cos(\angle ADC)$ is equal to
 (a) $\frac{7}{25}$ (b) $-\frac{7}{25}$
 (c) $\frac{24}{25}$ (d) $-\frac{24}{25}$
- In the ambiguous case if the remaining angles of a triangle with given a, b, A and B_1, B_2, C_1, C_2 then $\frac{\sin C_1}{\sin B_1} + \frac{\sin C_2}{\sin B_2} =$
 (a) $2 \cos A$ (b) $2 \sin B$
 (c) $2 \tan A$ (d) $2 \cot A$

- Two sides of a triangle are of lengths $\sqrt{6}$ and 4 and the angle opposite to smaller side is 30° , then how many such triangles are possible?
 (a) 0 (b) 1 (c) 2 (d) infinite
- If circumradius of triangle is 2, then the maximum value of $\frac{abc}{a+b+c}$ is
 (a) 1 (b) 2 (c) 3 (d) 4
- Let ABC and $AB'C$ be two non-congruent triangles with sides $BC = B'C = 5$, $AC = 6$, and $\angle A$ is fixed. If A_1 and A_2 are the areas of the two triangles ABC and $AB'C$, then the value of $\frac{A_1^2 + A_2^2 - 2A_1A_2 \cos 2A}{(A_1 + A_2)^2}$ is
 (a) $9/36$ (b) $25/36$ (c) $25/16$ (d) $16/25$

Multiple Correct Answers Type

- In triangle ABC , if $r_1 + r_2 = 3R$ and $r_2 + r_3 = 2R$, then
 (a) $\angle A = 90^\circ$ (b) $\angle B = 45^\circ$
 (c) $\angle C = 60^\circ$ (d) triangle ABC is right angled isosceles
- The radii r_1, r_2, r_3 of escribed circles of the triangle ABC are in harmonic progression. Let area of triangle ABC be 24 and its perimeter be 24. Then which of the following is(are) correct?
 (a) The length of longest side of triangle ABC is equal to 10
 (b) The radius of circle inscribed in triangle ABC is equal to 4.
 (c) The circumradius of triangle ABC is equal to 5.
 (d) The sides of triangle ABC are in A.P.

Answers Key

Single Correct Answer Type

1. (b) 2. (d) 3. (a) 4. (b) 5. (b)
 6. (a) 7. (c) 8. (d) 9. (b)

Multiple Correct Answers Type

10. (a, c) 11. (a, c, d)